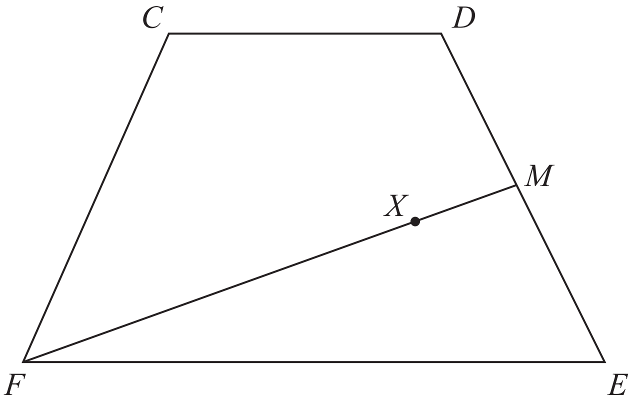
1. *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form. 1

$\overrightarrow{FE} = \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DE} = (\underline{a} - \underline{b}) + \underline{a} + \underline{b} = 2\underline{a}$

1

$2\underline{a}$ ....

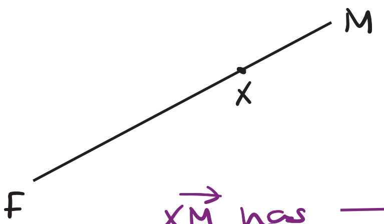
(2)

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$ $(n+1)$ parts.
CXE is a straight line.

(b) Work out the value of *n*.

$\overrightarrow{FM} = \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DM} = (\underline{a} - \underline{b}) + \underline{a} + \frac{1}{2}\underline{b}$

$= 2\underline{a} - \frac{1}{2}\underline{b}$. 1



$\overrightarrow{FX}$ has $\frac{n}{n+1}$ parts. $\therefore \overrightarrow{FX} = \frac{n}{n+1}\left(2\underline{a} - \frac{1}{2}\underline{b}\right)$

$\overrightarrow{XM}$ has $\frac{1}{n+1}$ parts. $\therefore \overrightarrow{XM} = \frac{1}{n+1}\left(2\underline{a} - \frac{1}{2}\underline{b}\right)$

CXE is a straight line $\therefore \overrightarrow{CX}$ is parallel to $\overrightarrow{CE}$.

$\overrightarrow{CX} = \overrightarrow{CF} + \overrightarrow{FX}$.

$\overrightarrow{CX} = -(\underline{a} - \underline{b}) + \frac{n}{n+1}\left(2\underline{a} - \frac{1}{2}\underline{b}\right)$

$n =$ 4 ....

(4)

(Total for Question is 6 marks)

$$\vec{CX} = (-\underline{a} + \underline{b}) + \frac{2n}{n+1}\underline{a} - \frac{n}{2(n+1)}\underline{b}$$

$$\vec{CX} = \left(-1 + \frac{2n}{n+1}\right)\underline{a} + \left(1 - \frac{n}{2(n+1)}\right)\underline{b}$$

$$\vec{CX} = \left(\frac{-(n+1) + 2n}{n+1}\right)\underline{a} + \left(\frac{2(n+1) - n}{2(n+1)}\right)\underline{b}.$$

$$\vec{CX} = \left(\frac{-n-1+2n}{n+1}\right)\underline{a} + \left(\frac{2n+2-n}{2(n+1)}\right)\underline{b}.$$

$$\vec{CX} = \left(\frac{n-1}{n+1}\right)\underline{a} + \left(\frac{n+2}{2(n+1)}\right)\underline{b}$$ (1)

$\vec{CX}$ is parallel to $\vec{CE}$, which mean $\vec{CX}$ is a multiple of $\vec{CE}$.

$\vec{CE} = \underline{a} + \underline{b}$. coefficient of $\underline{a}$ = coefficient of $\underline{b}$.

↳ same applies to $\vec{CX}$!

$$\therefore \frac{n-1}{\cancel{n+1}} = \frac{n+2}{2(\cancel{n+1})}.$$ (1)

$$n - 1 = \frac{n+2}{2}.$$

$\times 2$ $\times 2$

$$2(n-1) = n + 2.$$

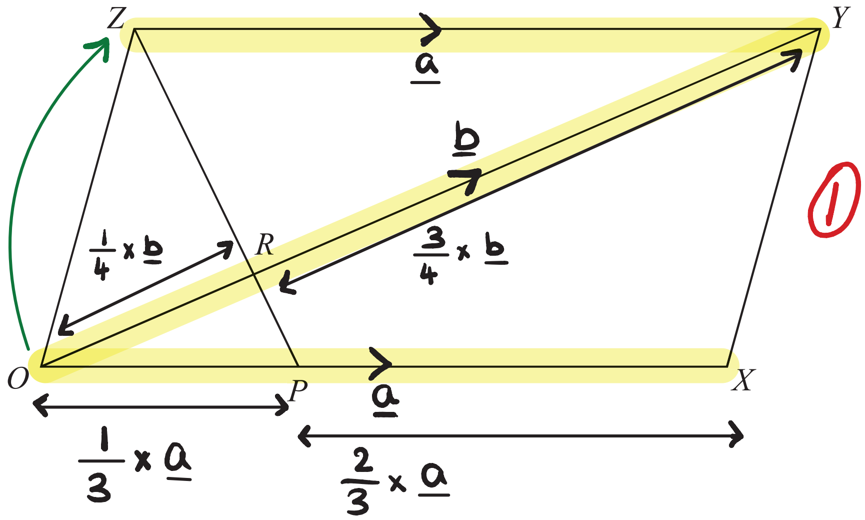
$$2n - 2 = n + 2.$$

$$n - 2 = 2.$$

$$\therefore n = 4.$$ (1)

2. $OXYZ$ is a parallelogram.

$\overrightarrow{ZY} = \underline{a}$
Since shape is a parallelogram



$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{OY} = \mathbf{b}$

P is the point on OX such that $OP : PX = 1 : 2$ $\quad 1 + 2 = 3$
R is the point on OY such that $OR : RY = 1 : 3$ $\quad 1 + 3 = 4$

Work out, in its simplest form, the ratio $ZP : ZR$
You must show all your working.

$$\overrightarrow{ZP} = \underline{a} - \underline{b} + \left(\frac{1}{3} \times \underline{a}\right)$$

$$\overrightarrow{ZR} = \underline{a} - \left(\frac{3}{4} \times \underline{b}\right) \quad (2)$$

$$\underline{a} - \underline{b} + \frac{\underline{a}}{3} : \underline{a} - \frac{3\underline{b}}{4}$$

$$\frac{3\underline{a}}{3} - \underline{b} + \frac{\underline{a}}{3} : \underline{a} - \frac{3\underline{b}}{4}$$

$$\frac{4\underline{a}}{3} - \underline{b} : \underline{a} - \frac{3\underline{b}}{4}$$

$$\frac{4\underline{a}}{3} - \underline{b} : \frac{3}{4}\left(\frac{4\underline{a}}{3} - \underline{b}\right)$$

$$(1) \quad \overrightarrow{ZP} : \frac{3}{4}(\overrightarrow{ZP})$$

$$\overrightarrow{ZR} = \frac{3}{4}\overrightarrow{ZP}$$

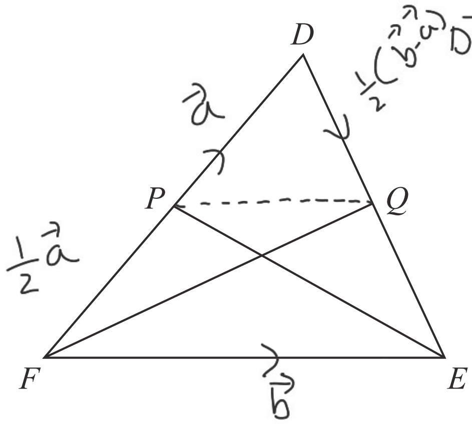
let $\overrightarrow{ZP} = 4$

$$\overrightarrow{ZR} = \frac{3 \times \cancel{4}}{\cancel{4}} \rightarrow \overrightarrow{ZR} = 3$$

Since we want $\overrightarrow{ZP} : \overrightarrow{ZR}$

4 : 3 (1)

3. *DEF* is a triangle.



$\overrightarrow{DE} = \overrightarrow{DF} + \overrightarrow{FE}$

$= -\vec{a} + \vec{b}$

$= \vec{b} - \vec{a}$

$\overrightarrow{DQ} = \frac{1}{2}\overrightarrow{DE}$

$= \frac{1}{2}(\vec{b} - \vec{a})$ ✓1

P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$

$= \frac{1}{2}\vec{a} + \frac{1}{2}(\vec{b} - \vec{a})$ ✓2

$= \cancel{\frac{1}{2}\vec{a}} + \frac{1}{2}\vec{b} - \cancel{\frac{1}{2}\vec{a}}$

$= \frac{1}{2}\vec{b}$ ✓3

for 2 vectors to be parallel, they must be scalar multiples of eachother.

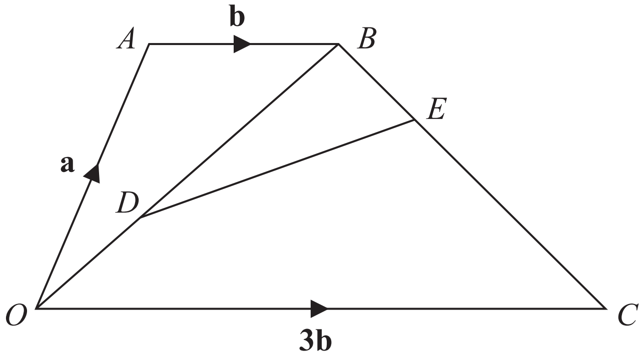
$\overrightarrow{PQ} = \frac{1}{2}\vec{b}$ and $\overrightarrow{FE} = \vec{b}$

$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{FE}$ and so $\overrightarrow{PQ}$ is a scalar multiple of $\overrightarrow{FE}$.

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{FE}$ are parallel, as required. ✓4

(Total for Question is 4 marks)

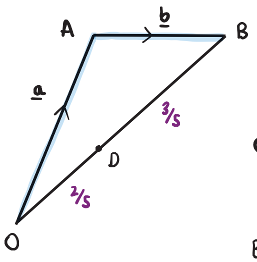
4. $OABC$ is a trapezium.



$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{AB} = \mathbf{b}$
$\overrightarrow{OC} = 3\mathbf{b}$

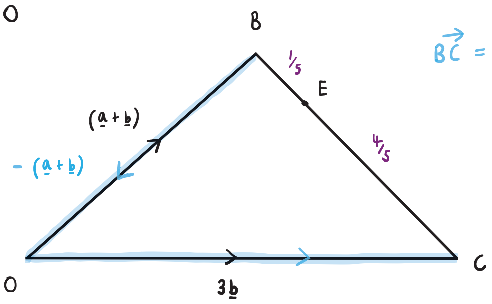
D is the point on OB such that $OD : DB = 2 : 3$
E is the point on BC such that $BE : EC = 1 : 4$

Work out the vector $\overrightarrow{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.



$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \underline{a} + \underline{b}$ (1)

$OD : DB = 2 : 3$ total 5 parts.

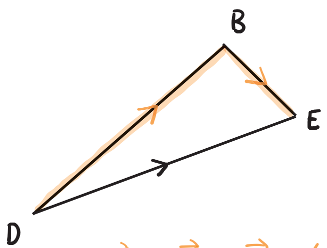


$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -(\underline{a} + \underline{b}) + 3\underline{b}$

$= -\underline{a} - \underline{b} + 3\underline{b} = -\underline{a} + 2\underline{b}$

$BE : EC = 1 : 4$ total 5 parts (1)

NOTE : $\overrightarrow{BO} = -(\overrightarrow{OB})$!



$\overrightarrow{DB} = \frac{3}{5}(\overrightarrow{OB}) = \frac{3}{5}(\underline{a} + \underline{b}) = \frac{3}{5}\underline{a} + \frac{3}{5}\underline{b}$

$\overrightarrow{BE} = \frac{1}{5}(\overrightarrow{BC}) = \frac{1}{5}(-\underline{a} + 2\underline{b}) = -\frac{1}{5}\underline{a} + \frac{2}{5}\underline{b}$ (1)

$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE} = \left(\frac{3}{5}\underline{a} + \frac{3}{5}\underline{b}\right) + \left(-\frac{1}{5}\underline{a} + \frac{2}{5}\underline{b}\right)$ (1)

$= \frac{2}{5}\underline{a} + \underline{b}$

$\frac{2}{5}\underline{a} + \underline{b}$

(Total for Question is 4 marks)